

$$\max c^T x$$

$$Ax = b$$

$$x \geq 0$$

$$\Rightarrow (B \ S) \cdot \begin{pmatrix} x_B \\ x_S \end{pmatrix} = b \Rightarrow$$

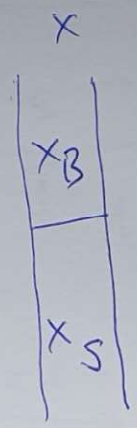
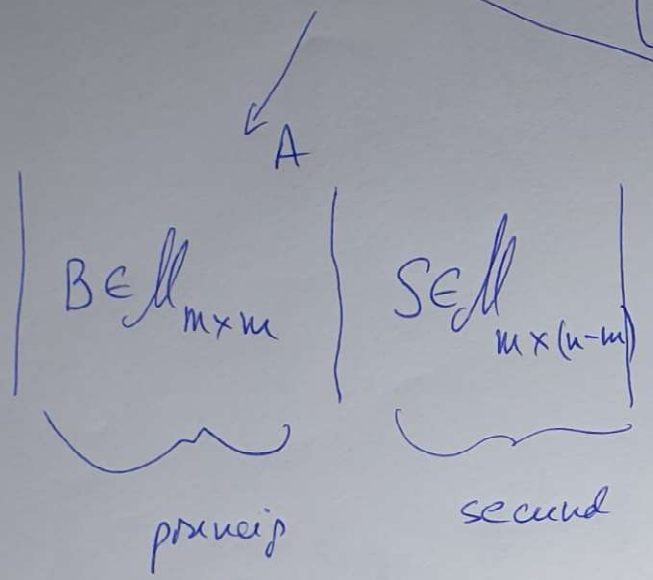
$$\Rightarrow Bx_B + Sx_S = b$$

$$\Downarrow$$

$$Bx_B = b - Sx_S \Rightarrow$$

$$\Rightarrow x_B = \bar{B}^{-1}b - \bar{B}^{-1}Sx_S$$

$$\Downarrow$$

$$x_B$$


$$d(x) = c \cdot x = (c_B \ c_S) \cdot \begin{pmatrix} x_B \\ x_S \end{pmatrix} =$$

$$= \bar{c}_B \cdot x_B + \bar{c}_S \cdot x_S = c_B (\bar{B}^{-1}b - \bar{B}^{-1}Sx_S) + c_S x_S =$$

$$= \underbrace{c_B \bar{B}^{-1}b}_{d(x_B)} - c_B \cdot \bar{B}^{-1}Sx_S + c_S x_S$$

$$f(x) = f(x_B) + (c_S - c_B \cdot B^{-1} \cdot S) x_S$$

$f(x_B)$  maximum für  $(\forall) x_S \geq 0 \Rightarrow$

$$\Rightarrow c_S - c_B \cdot B^{-1} \cdot S \leq 0$$

$\underbrace{\hspace{10em}}_{\Delta_S} \quad \underbrace{\hspace{10em}}_{z_S}$

$$c_B \cdot B^{-1} \cdot S = \left( \overset{c_B}{\longleftarrow} \quad \left| \begin{array}{c} | \\ | \\ | \\ | \end{array} \right. \cdot \left| \begin{array}{c} | \\ | \\ | \\ | \end{array} \right. \right) \cdot B^{-1} \cdot S \left| \begin{array}{c} | \\ | \\ | \\ | \end{array} \right.$$

$x_B$   
 $x_B$  solutia de baza  $\geq 0$   $-B^{-1} \cdot S \cdot x_N$

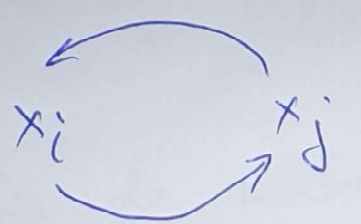
$$x_1 = x_1 + a_{1m+1} x_{m+1} + \dots + a_{1j} x_j + \dots + a_{1n} x_n$$


---


$$x_i = x_i + a_{im+1} x_{m+1} + \dots + \boxed{a_{ij} x_j} + \dots + a_{in} x_n$$


---


$$x_m = x_m + a_{mm+1} x_{m+1} + \dots + a_{mj} x_j + \dots + a_{mn} x_n = 0$$



①  $a_{ij} \neq 0$

$$f(x) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n =$$

$$= \underbrace{c_m (x_1 + a_{1m+1} x_{m+1} + \dots + a_{1n} x_n)}_m + \underbrace{\dots}_{\text{dar ultimele } n-m}$$


---


$$c_{m+1} x_{m+1} + \dots + c_n x_n$$

$n-m$

$$f(x) = \underbrace{c_1 x_1 + \dots + c_m x_m}_{f(x_B)} + \underbrace{(c_{m+1} - c_1 a_{1, m+1} - c_2 a_{2, m+1} - \dots)}_{\Delta_{m+1}} x_{m+1}$$

$$x_{m+1} \rightarrow c_{m+1} - (c_1, c_2, \dots, c_m) \begin{pmatrix} a_{1, m+1} \\ a_{2, m+1} \\ \vdots \\ a_{m, m+1} \end{pmatrix}$$

$$z_{m+1}$$

$$\Delta_{m+1}$$

$$\Rightarrow f(x) = f(x_B) + \sum \Delta_{m+1} x_{m+1}$$

$x_B =$  solutia de maxim  $\Rightarrow$  \*

$$\Rightarrow f(x) \leq f(x_B) \quad (\forall) x \geq 0 \Rightarrow \underbrace{\sum \Delta_{m+1} x_{m+1} \leq 0}_{\substack{(\forall) x_{m+1} \\ \vdots \\ x_n}} > 0$$

$$\Rightarrow \boxed{\Delta_{m+1} = c_{m+1} - z_{m+1} \leq 0} \quad (\forall) m+1, \dots, n$$

= conditia ca o solutie de baza  
sa fie optima



$$x_j = \left( -\frac{x_i}{a_{ij}} \right)^{\geq 0} - \frac{a_{i,m+1}}{a_{ij}} x_{m+1} \dots + \frac{1}{a_{ij}} x_i - \dots - \frac{a_{i,n}}{a_{ij}} x_n$$

$$\Rightarrow a_{ij} \geq 0 \quad \text{pr} \quad \bar{c}_i \geq 0$$

②

$$x_k \neq j = \left( x_k - a_{kj} \frac{x_i}{a_{ij}} \right)$$

$$(*) \quad k \in \{1, \dots, m\} \\ k \neq j$$

(3)

$\Downarrow$

$$x_k - a_{kj} \cdot \frac{x_i}{a_{ij}} \geq 0 \quad (*) k$$

stim deja că

$$\left. \begin{array}{l} x_i \geq 0 \\ x_k \geq 0 \\ a_{ij} > 0 \end{array} \right\} \text{sol inițială} \\ \text{era pozitivă} \quad (2)$$

a) dacă  $a_{kj} \leq 0 \Rightarrow$  tot e pozitiv

dacă  $a_{kj} > 0$

$$\Rightarrow \frac{x_k}{a_{kj}} \geq \frac{x_i}{a_{ij}} \quad (*) k \neq j \quad (3) \Rightarrow$$

$\Rightarrow \frac{x_i}{a_{ij}}$  e cel mai mic raport dintre cele pozitive

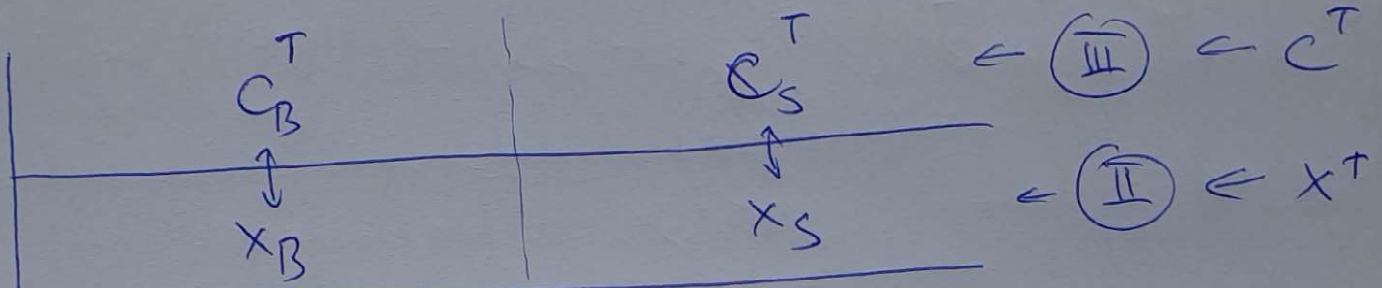
$f(x_i) \leftrightarrow f(x_j)$  vs  $f(x_i)$

$$f(x_i) = c_1 x_1 + \dots + c_m x_m + \sum_{m+1}^{n+1} \lambda_{m+1} x_{m+1}$$

$$f(x_j) = \begin{matrix} 0 \\ \uparrow \end{matrix} \quad \begin{matrix} \uparrow \\ -\frac{x_i}{a_{ij}} \Delta_j \end{matrix}$$

$$\left. \begin{matrix} -\frac{x_i}{a_{ij}} \Delta_j \geq 0 \\ \Rightarrow \text{lag ael} \\ \Delta_j \leq 0 \text{ pkr} \end{matrix} \right\} \text{ core } \frac{x_i}{a_{ij}} |\Delta_j| = \underline{\underline{\text{maxim}}} \quad x_j$$

# SIMPLEX



V	IV	III
$\downarrow$	$\downarrow$	$\downarrow$
$C_B$	$X_B$	$Z_B$ = $B^{-1} \cdot b$
		$f(X_B)$

$\textcircled{\text{I}} = B^{-1} \cdot A$

$B^{-1} \cdot X_S$

$I_m$

$Z = C_B^T \cdot B^{-1} \cdot A$

$Z - C$  ou  $C - Z$

$\leftarrow \textcircled{\text{VII}} \neq$

$\leftarrow \textcircled{\text{VIII}} \Delta$

$C_B^T \cdot B^{-1} \cdot b$

la algebre

la  $Z - C \rightarrow$  optim  
ou  
toti  $\geq 0$

la  $C - Z \rightarrow$  toti  $\leq 0$