$\boxed{\max c^T x}$

$$\boxed{Ax = b} \qquad \boxed{x \geq 0}$$

$$A = \Big(\underbrace{B \in \mathcal{M}_{m \times m}}_{\text{princip}} \;\Big|\; \underbrace{S \in \mathcal{M}_{m \times (n-m)}}_{\text{secund}}\Big) \qquad x = \begin{pmatrix} x_B \\ x_S \end{pmatrix}$$

$$\Rightarrow (B\ S) \cdot \begin{pmatrix} x_B \\ x_S \end{pmatrix} = b \Rightarrow$$

$$\Rightarrow B x_B + S x_S = b$$

$$\Downarrow$$

$$B x_B = b - S x_S \Rightarrow$$

$$\Rightarrow \boxed{x_B = B^{-1} b - B^{-1} S x_S}$$

$$\parallel$$

$$\mathfrak{x}_B$$

$$f(x) = c \cdot x = (c_B\ c_S) \cdot \begin{pmatrix} x_B \\ x_S \end{pmatrix} =$$

$$= \bar{c}_B \cdot x_B + \bar{c}_S\, x_S = c_B \left(B^{-1} b - B^{-1} S x_S \right) + c_S x_S =$$

$$= \underbrace{c_B \overbrace{B^{-1} \cdot b}^{\mathfrak{x}_B}}_{f(\mathfrak{x}_B)} - c_B \cdot B^{-1} S x_S + c_S x_S$$

$$f(x) = f(x_B) + (c_S - c_B \cdot B^{-1} \cdot S)\, x_S$$

$f(x_B)$ maxim pt $(\forall)\ x_S \geq 0 \implies$

$$\implies \boxed{\underbrace{c_S - \underbrace{c_B \cdot B^{-1} \cdot S}_{= z_S}}_{= \Delta_S}} \leq 0$$

$$c_B \cdot B^{-1} \cdot S = \left(\xleftrightarrow{\quad c_B \quad} \right) \cdot \left(\Big| \ \Big| \quad B^{-1} \cdot S \quad \Big| \ \Big| \ \Big| \right)$$

$x_B = \mathfrak{x}_B$ soluția de bază ≥ 0

$$x_1 = \mathfrak{x}_1 + \overbrace{a_{1m+1}x_{m+1} + \dots + a_{1j}x_j + \dots + a_{1n}x_n}^{-B^{-1}\cdot S\cdot x_S}$$

$$x_i = \mathfrak{x}_i + a_{im+1}x_{m+1} + \dots + \boxed{a_{ij}x_j} + \dots + a_{in}x_n$$

$$x_m = \mathfrak{x}_m + a_{mm+1}x_{m+1} + \dots + a_{mj}x_j + \dots + a_{mn}x_n$$

$= 0$

$$x_i \rightleftarrows x_j$$

① $a_{ij} \neq 0$

$$f(x) = c_1\cancel{x_1} + c_2\cancel{x_2} + \dots + c_n\cancel{x_n} =$$

$$= \left.\begin{array}{l} c_1\left(\mathfrak{x}_1 + \underbrace{a_{1m+1}x_{m+1} + \dots + a_{1n}x_n}_{\text{doar ultimele } n-m}\right) \\ + \\ \vdots \\ + \end{array}\right\} m$$

$$\underbrace{c_{m+1}x_{m+1} + \dots + c_n x_n}_{n-m}$$

$$f(x) = \underbrace{c_1 x_1 + \dots + c_m x_m}_{f(x)} + (\underbrace{c_{m+1} - c_1 a_{1\,m+1} - c_2 a_{2\,m+1} - \dots}\,)\, x_{m+1}$$

$$x_{m+1} \longrightarrow \underbrace{c_{m+1} - \underbrace{(c_1\ c_2\ \dots\ c_m)\begin{pmatrix} a_{1\,m+1} \\ a_{2\,m+1} \\ \vdots \\ a_{m\,m+1} \end{pmatrix}}_{z_{m+1}}}_{\Delta_{m+1}}$$

$$\Rightarrow f(x) = f(x_B) + \sum \Delta_{m+1} x_{m+1}$$

x_B = soluția de maxim $\Rightarrow$ $\underbrace{\sum \Delta_{m+1} x_{m+1} \le 0}$ $(\forall)\ \left.\begin{matrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ x_n \end{matrix}\right\} \ge 0$

$\Rightarrow f(x) \le f(x_B)\ (\forall)\ x \ge 0 \Rightarrow$

$\Rightarrow \boxed{\Delta_{m+1} = c_{m+1} - z_{m+1} \le 0}$ $(\forall)\ m+1, \dots, n$

= condiția ca o soluție de bază să fie optimă

$$
\begin{tikzcd}
x_i \arrow[r, bend right] & x_j \arrow[l, bend right]
\end{tikzcd}
$$

$$x_j = \overbrace{-\frac{x_i}{a_{ij}}}^{\geq 0} - \frac{a_{im+1}}{a_{ij}} x_{m+1} \; --- + \frac{1}{a_{ij}} x_i - \; -- - \frac{a_{in}}{a_{ij}} x_n$$

$\Rightarrow$ $a_{ij} \geq 0$ phr cā $x_i \geq 0$

2

$$x_{\alpha \neq j} = x_\alpha - a_{\alpha j} \frac{x_i}{a_{ij}} \quad (\forall)\ \alpha \in \{1, \dots, m\},\ \alpha \neq j$$

(3)

$\Downarrow$

$$\boxed{x_\alpha - a_{\alpha j} \cdot \frac{x_i}{a_{ij}} \geq 0 \quad (\forall)\alpha}$$

Stim deja că

$x_i \geq 0$
$x_\alpha \geq 0$ } sol inițiala era pozitivă
$a_{ij} > 0$ (2)

a) dacă $a_{\alpha j} \leq 0 \Rightarrow$ tot e pozitiv

daca $\boxed{a_{\alpha j} > 0} \Rightarrow \boxed{\frac{x_\alpha}{a_{\alpha j}} \geq \frac{x_i}{a_{ij}} \quad (\forall)\alpha \neq j}$ (3) $\Rightarrow$

$\Rightarrow \frac{x_i}{a_{ij}}$ e cel mai mic raport dintre cele pozitive

$$f(x_i \ \ x_j) \text{ vs } f(x_i)$$

$$f(x_i) = c_1 x_1 + \dots + c_m x_m + \sum y_{m+1} x_{m+1}$$

$$f(x_j) = \qquad 0 \qquad\qquad\qquad -\frac{x_i}{a_{ij}} \Delta_j$$

$$-\frac{x_i}{a_{ij}} \Delta_j \geq 0$$

$\Rightarrow$ Aleg acel x_j

$\Delta_j \leq 0$ pt

care $\dfrac{x_i}{a_{ij}} |\Delta_j| =$ <u>maxim</u>

SIMPLEX

			c_B^T	c_S^T
			x_B	x_S
V	IV	VI		
c_B	x_B	$x_B = B^{-1} \cdot b$	$\text{I} = B^{-1} \cdot A$: I_m	$B^{-1} \cdot \cancel{X}_S$
		$f(x_B)$	$z = c_B^T \cdot B^{-1} \cdot A$	
			$z - c$ sau $c - z$	

← III ← c^T

← II ← x^T

← VII z

← VIII Δ

$c_B^T \cdot B^{-1} \cdot b$

la alegere

la $z - c$ → optim ⇔ toți ≥ 0

la $c - z$ → toți ≤ 0