Se da problema de transport:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Destinatii****Surse** | **D1** | **D2** | **D3** | **D4** | **D5** | **Disponibil** |
| **S1** | 5 | 1 | 8 | 7 | 5 | **15** |
| **S2** | 3 | 9 | 6 | 7 | 8 | **25** |
| **S3** | 4 | 2 | 7 | 6 | 5 | **42** |
| **S4** | 7 | 11 | 10 | 4 | 9 | **35** |
| **Cerere** | **30** | **20** | **15** | **10** | **20** |  |

1. Verificati daca este echilibrata
2. Echilibrati problema, daca nu este
3. Verificati daca problema admite solutii degenerate
4. Gasiti cate o solutie initiala prin metodele cunoscute (NV,C­minL,CminC,Cmin­,DM)
5. Gasiti solutia optima.
6. Echilibrata ⬄ suma cererilor = suma disponibilurilor

Suma cererilor = 30+20+15+10+20 = 95

Suma disponibilurilor = 15+25+42+35 = 117

Cum cele 2 sume sunt diferite rezulta ca problema NU e echilibrata

1. Deoarece oferta este mai mare decat cererea, echilibrarea se face prin adaugarea unei destinatii suplimentare a carei cere e fix surplusul de oferta (117 – 95 = 22):

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Destinatii****Surse** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **Disponibil** |
| **S1** | 5 | 1 | 8 | 7 | 5 | 0 | **15** |
| **S2** | 3 | 9 | 6 | 7 | 8 | 0 | **25** |
| **S3** | 4 | 2 | 7 | 6 | 5 | 0 | **42** |
| **S4** | 7 | 11 | 10 | 4 | 9 | 0 | **35** |
| **Cerere** | **30** | **20** | **15** | **10** | **20** | **22** |  |

Transporturile finale de la surse spre aceasta destinatie vor fi cantitatile care raman la surse si, pentru ca acest transport nu se face, de fapt si nici nu trebuie sa influenteze rezultatul, costurile unitare pe aceste rute vor fi 0.

1. Problema are solutii degenerate daca si numai daca exista o submultime stricta si nevida de surse si o submultime stricta si nevida de destinatii cu proprietatea ca oferta totala a submultimii de surse este egala cu cererea totala a submultimii de destinatii. Se observa ca, de exemplu: S1+S2=D2+D5, deci problema **admite** solutii degenerate.
2. Gasirea unei solutii initiale
	1. Metoda Nord-Vest

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 15 |  |  |  |  |  | **15-15** |
| 15 | 10 |  |  |  |  | **25-15-10** |
|  | 10 | 15 | 10 | 7 |  | **42-10-15-10-7** |
|  |  |  |  | 13 | 22 | **35-13-22** |
| **30-15-15** | **20-10-10** | **15-15** | **10-10** | **20-7-13** | **22-22** |  |

Avem 9 rute = 4+6-1 deci solutia e nedegenerata

Cost total = 15\*5+15\*3+10\*9+10\*2+15\*7+10\*6+7\*5+13\*9+22\*0 = 547

* 1. Metoda costului minim pe linii

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | 15 | **15-15** |
| 18 |  |  |  |  | 7 | **25-7-18** |
| 12 | 20 |  |  | 10 |  | **42-20-12-10** |
|  |  | 15 | 10 | 10 |  | **35-10-15-10** |
| **30-18-12** | **20-20** | **15** | **10** | **20-10** | **22-15-7** |  |

Avem 9 rute = 4+6-1 deci solutia e nedegenerata

Cost total = 15\*0+18\*3+7\*0+12\*4+20\*2+10\*5+15\*10+10\*4+10\*9 = 472

* 1. Metoda costului minim pe coloane

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 15 |  |  |  |  | **15-15** |
| 25 |  |  |  |  |  | **25-25** |
| 5 | 5 | 15 |  | 17 |  | **42-5-5-15-17** |
|  |  |  | 10 | 3 | 22 | **35-10-3-22** |
| **30-25-5** | **20-15-5** | **15-15** | **10-10** | **20-17-3** | **22-22** |  |

Avem 9 rute = 4+6-1 deci solutia e nedegenerata

Cost total = 15\*1+25\*3+5\*4+5\*2+15\*7+17\*5+10\*4+3\*9+22\*0 = 377

* 1. Metoda costului minim din tot tabelul

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | 15 | **15-15** |
| 18 |  |  |  |  | 7 | **25-7-18** |
| 12 | 20 |  |  | 10 |  | **42-20-12-10** |
|  |  | 15 | 10 | 10 |  | **35-10** |
| **30-18-12** | **20-20** | **15** | **10-10** | **20-10** | **22-15-7** |  |

Se observa ca este aceeasi solutie ca la costul minim pe linii, deci nedegenerata si cu costul total = 472

* 1. Metoda diferentelor maxime

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 1 | 1 | 2 | 0 | 0 |  |
| 1 |  |  |  |  |  |  | **15** |
| 3 |  |  |  |  |  |  | **25** |
| 2 |  |  |  |  |  |  | **42** |
| **4** |  |  |  |  |  | **22** | **35-22** |
|  | **30** | **20** | **15** | **10** | **20** | **22-22** |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 1 | 1 | 2 | 0 |  |  |
| **4** |  | **15** |  |  |  |  | **15-15** |
| 3 |  |  |  |  |  |  | **25** |
| 2 |  |  |  |  |  |  | **42** |
| 3 |  |  |  |  |  | 22 | **35-22** |
|  | **30** | **20-15** | **15** | **10** | **20** | **22-22** |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | **7** | 1 | 2 | 3 |  |  |
|  |  | 15 |  |  |  |  | **15-15** |
| 3 |  |  |  |  |  |  | **25** |
| 2 |  | **5** |  |  |  |  | **42-5** |
| 3 |  |  |  |  |  | 22 | **35-22** |
|  | **30** | **20-15-5** | **15** | **10** | **20** | **22-22** |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 |  | 1 | 2 | **3** |  |  |
|  |  | 15 |  |  |  |  | **15-15** |
| **3** | **25** |  |  |  |  |  | **25-25** |
| 1 |  | 5 |  |  |  |  | **42-5** |
| **3** |  |  |  |  |  | 22 | **35-22** |
|  | **30-25** | **20-15-5** | **15** | **10** | **20** | **22-22** |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 3 |  | 3 | 2 | **4** |  |  |
|  |  | 15 |  |  |  |  | **15-15** |
|  | 25 |  |  |  |  |  | **25-25** |
| 1 |  | 5 |  |  | **20** |  | **42-5-20** |
| 3 |  |  |  |  |  | 22 | **35-22** |
|  | **30-25** | **20-15-5** | **15** | **10** | **20-20** | **22-22** |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **3** |  | **3** | 2 |  |  |  |
|  |  | 15 |  |  |  |  | **15-15** |
|  | 25 |  |  |  |  |  | **25-25** |
| 2 | **5** | 5 |  |  | 20 |  | **42-5-20-5** |
| **3** |  |  |  |  |  | 22 | **35-22** |
|  | **30-25-5** | **20-15-5** | **15** | **10** | **20-20** | **22-22** |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 |  |  |  |
|  |  | 15 |  |  |  |  | **15-15** |
|  | 25 |  |  |  |  |  | **25-25** |
| 1 | 5 | 5 |  |  | 20 |  | **42-5-20-5** |
| **6** |  |  |  | **10** |  | 22 | **35-22-10** |
|  | **30-25-5** | **20-15-5** | **15** | **10-10** | **20-20** | **22-22** |  |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 15 |  |  |  |  | **15-15** |
| 25 |  |  |  |  |  | **25-25** |
| 5 | 5 | **12** |  | 20 |  | **42-5-20-5-12** |
|  |  | **3** | 10 |  | 22 | **35-22-10-3** |
| **30-25-5** | **20-15-5** | **15-12-3** | **10-10** | **20-20** | **22-22** |  |

Avem 9 rute = 4+6-1 deci solutia e nedegenerata

Cost total = 15\*1+25\*3+5\*4+5\*2+12\*7+20\*5+3\*10+10\*4+22\*0 = 374

1. Gasirea solutiei optime (Obs. In mod normal se porneste de la solutia de cost minim, dar, in acest caz, pentru a fi siguri ca va exista si o imbunatatire a solutiei, vom porni de la solutia obtinuta prin metoda costului minim pe coloane care este sigur suboptimala)
	1. Introducerea variabilelor u si v:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Destinatii****Surse** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** |  |
| **S1** |  | 15(c=1) |  |  |  |  | **u1** |
| **S2** | 25(c=3) |  |  |  |  |  | **u2** |
| **S3** | 5(c=4) | 5(c=2) | 15(c=7) |  | 17(c=5) |  | **u3** |
| **S4** |  |  |  | 10(c=4) | 3(c=9) | 22(c=0) | **u4** |
|  | **v1** | **v2** | **v3** | **v4** | **v5** | **v6** |  |

* 1. Construirea sistemului ui+vj=cij (cate o ecuatie pentru fiecare ruta folosita in solutie)

u1+v2=1

u2+v1=3

u3+v1=4

u3+v2=2

u3+v3=7

u3+v5=5

u4+v4=4

u4+v5=9

u4+v6=0

* 1. Gasirea rapida a unei solutii particulare a sistemului (avem o necunoscuta in plus, deci o infinitate de solutii, dara avem nevoie de o solutie particulara pe care sa o gasim cat mai usor si rapid.): vom egala cu 0 variabila care apare de cele mai multe ori (u3)

u3=0 => v1=4, v2=2,v3=7,v5=5

v1=4 => u2=-1

v2=2 => u1=-1

v5=5 => u4=4

u4=4 => v4=0, v6=-4

* 1. Calcularea Δij = ui+vj - cij pentru rutele care nu fac parte din solutie:

Δ11 = -1+4-5 = -2

Δ13 = -1+7-8 = -2

Δ14 = -1+0-7 = -8

Δ15 = -1+5-5 = -1

Δ16 = -1-4-0 = -5

Δ22 = -1+2-9 = -8

Δ23 = -1+7-6 = 0

Δ24 = -1+0-7 = -8

Δ25 = -1+5-8 = -4

Δ26 = -1-4-0 = -5

Δ34 = 0+0-6 = -6

Δ36 = 0-4-0 = -4

Δ41 = 4+4-7 = 1

Δ42 = 4+2-11 = -5

Δ43 = 4+7-10 = 1

Deoarece exista Δ43 strict pozitivi => solutia nu este optima. Va intra in baza ruta care are cel mai pozitiv Δ. Cum in acest caz maximul este multiplu alegem unul la intamplare (Δ41 lexicografic primul).

* 1. Gasirea circuitului care pleaca din ruta (S4 -> D1) si trece doar prin rute din solutie mergand doar pe orizontala si verticala si schimband directia la fiecare pas

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Destinatii****Surse** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** |  |
| **S1** |  | 15(c=1) |  |  |  |  | **u1** |
| **S2** | 25(c=3) |  |  |  |  |  | **u2** |
| **S3** | 5(c=4) | 5(c=2) | 15(c=7) |  | 17(c=5) |  | **u3** |
| **S4** |  |  |  | 10(c=4) | 3(c=9) | 22(c=0) | **u4** |
|  | **v1** | **v2** | **v3** | **v4** | **v5** | **v6** |  |

Se observa ca acest traseu este unic

* 1. Notam alternativ cu + si – rutele de pe traseu incepand cu + din ruta care va intra in solutie

+

+

-

-

* 1. Minimul dintre cantitatile transportate pe rutele cu – este θ = 3, iar acesta se scade din transporturile pe rutele cu – si se adauga la cele cu + rezultand noua solutie:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Destinatii****Surse** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** |  |
| **S1** |  | 15(c=1) |  |  |  |  | **u1** |
| **S2** | 25(c=3) |  |  |  |  |  | **u2** |
| **S3** | 2(c=4) | 5(c=2) | 15(c=7) |  | 20(c=5) |  | **u3** |
| **S4** | **3(c=7)** |  |  | 10(c=4) | **0**(c=9) | 22(c=0) | **u4** |
|  | **v1** | **v2** | **v3** | **v4** | **v5** | **v6** |  |

A intrat ruta S4->D1 si a iesit ruta S4->D5, micsorarea costului fiind de Δ41\* θ = 3 deci noul cost este 374 (care este la fel de mic ca cel de la metoda diferentelor maxime). Reluam algoritmul de la pasul 2:

2. sistemul ui+vj=cij

u1+v2=1

u2+v1=3

u3+v1=4

u3+v2=2

u3+v3=7

u3+v5=5

u4+v1=7

u4+v4=4

u4+v6=0

3. Solutia sistemului (alegem tot u3=0)

u3=0 => v1=4, v2=2,v3=7,v5=5

v1=4 => u2=-1, u4=3

v2=2 => u1=-1

u4=3 => v4=1, v6=-3

4. Calcularea Δij = ui+vj - cij pentru rutele care nu fac parte din solutie:

Δ11 = -1+4-5 = -2

Δ13 = -1+7-8 = -2

Δ14 = -1+1-7 = -7

Δ15 = -1+5-5 = -1

Δ16 = -1-3-0 = -4

Δ22 = -1+2-9 = -8

Δ23 = -1+7-6 = 0

Δ24 = -1+1-7 = -7

Δ25 = -1+5-8 = -4

Δ26 = -1-3-0 = -4

Δ34 = 0+1-6 = -5

Δ36 = 0-3-0 = -3

Δ42 = 3+2-11 = -6

Δ43 = 3+7-10 = 0

Δ45 = 3+5-9 = -1

Toti Δij sunt <= 0, deci solutia este optima.